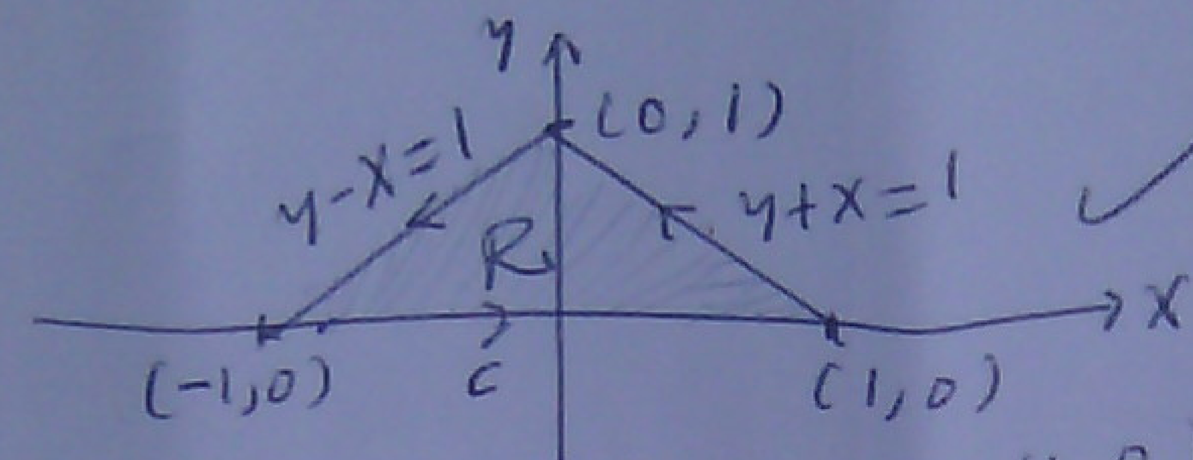
3. Use **Green's Theorem** to evaluate $\int_C (e^x + y^2)dx + (e^y + x^2)dy$, where C is the boundary of the triangle with vertices (-1, 0), (1, 0), (0, 1)



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• Equation of line segment from (1,0) to (0,1):

$\frac{y-0}{x-1} = \frac{1-0}{0-1} \Rightarrow y = 1-x$ 1

• Equation of line segment from (0,1) to (-1,0):

$\frac{y-0}{x-0} = \frac{0-1}{-1-0} \Rightarrow y = x+1$ 1

The region $R = \{(x,y): y-1 \le x \le 1-y,\ 0 \le y \le 1\}$ which is horizontally simple

Green's Theorem $\Rightarrow$

$\int_C (e^x + y^2)dx + (e^y + x^2)dy = \iint_R [\frac{\partial}{\partial x}(e^y + x^2) - \frac{\partial}{\partial y}(e^x + y^2)]dA$ 1

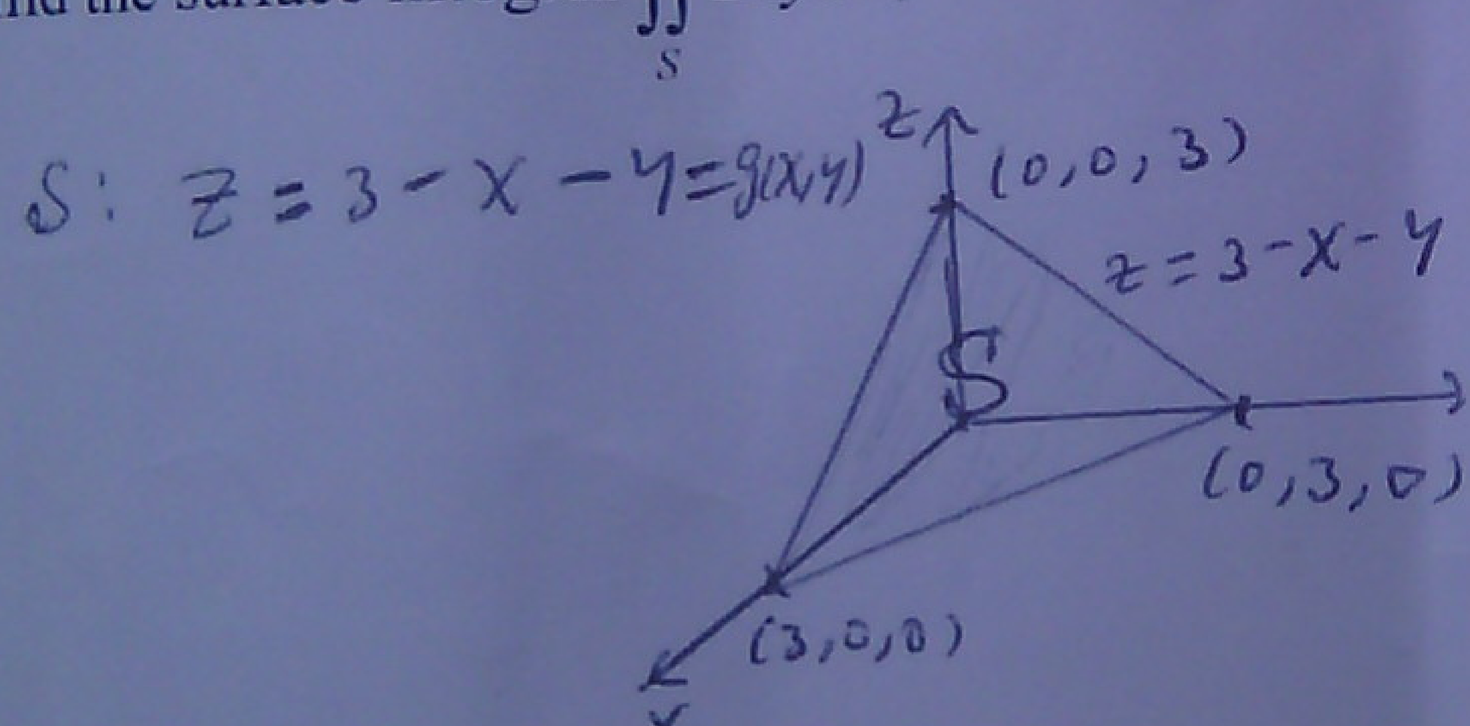
$= \int_0^1 \int_{y-1}^{1-y} (2x - 2y)\,dx\,dy$

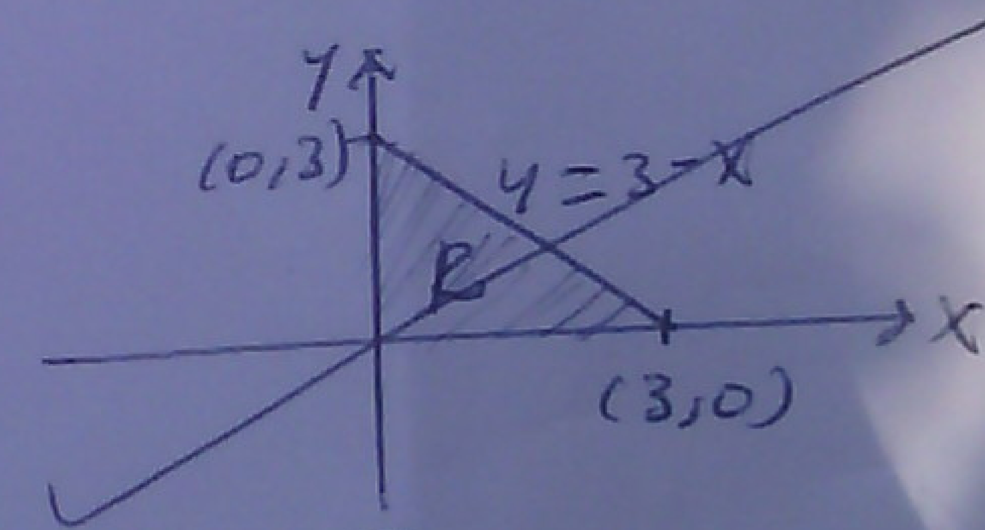
$= \int_0^1 (x^2 - 2yx)\Big|_{y-1}^{1-y} dy$

$= \int_0^1 (4y^2 - 4y)\,dy = \frac{4y^3}{3} - 2y^2 \Big|_0^1 = -\frac{2}{3}$ 1

4. Find the surface integral $\iint_S 2xy\,ds$, where the surface $S: z = 3 - x - y$, in the first octant.

$S: z = 3 - x - y = g(x,y)$





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$\Rightarrow g_x = -1,\quad g_y = -1$ 1

$\sqrt{g_x^2 + g_y^2 + 1^2} = \sqrt{(-1)^2 + (-1)^2 + 1^2} = \sqrt{3}$ 1

The region $R = \{(x,y): 0 \le y \le 3-x,\ 0 \le x \le 3\}$ which is vertically simple.

surface integral

$\iint_S 2xy\,dS = \iint_R 2xy\sqrt{3}\,dA$ 1

$= \int_0^3 \int_0^{3-x} \sqrt{3}\,2xy\,dy\,dx$

$= \int_0^3 2\sqrt{3}\,x\,\frac{y^2}{2}\Big|_0^{3-x} dx$

$= \int_0^3 \sqrt{3}\,x(9 - 6x + x^2)dx$

$= \sqrt{3}\left(\frac{9x^2}{2} - \frac{6x^3}{3} + \frac{x^4}{4}\right)\Big|_0^3 = \sqrt{3}\left(\frac{81}{2} - 54 + \frac{81}{4}\right) = \frac{27}{4}\sqrt{3}$ 1